Eigenstructure Assignment with Gain Suppression Using Eigenvalue and Eigenvector Derivatives

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Eigenstructure assignment is considered with gain suppression in which selected entries in the output feedback gain matrix are eliminated. This approach may yield a simpler controller with only a small effect on performance. Early results used partial derivatives of the eigenvalues with respect to the gains in order to determine which entries of the feedback gain matrix should be selected for elimination. In this paper, expressions for the partial derivatives of the eigenvectors with respect to the feedback gains are derived and used to compute the expected shift in the eigenvectors if the *ij*th feedback gain were eliminated. This establishes an improved design method for eigenstructure assignment with gain suppression by a priori choosing to eliminate those gains that have the smallest influence on both the eigenvalues and eigenvectors. An example of a fighter aircraft is presented that illustrates the importance of using both eigenvalue and eigenvector derivative information when choosing which feedback gains should be eliminated.

Introduction

EIGENSTRUCTURE assignment is a design approach that incorporates classical specifications on damping, settling time, and mode decoupling into a modern multivariable control framework. This approach was used by Andry et al.[1] to synthesize a control law for the linearized lateral dynamics of the L-1011 aircraft. Reference 1 also proposed the elimination of selected entries in the output feedback gain matrix, which is called gain suppression. The use of gain suppression may yield a simpler controller with only a small effect on performance. Reference 1 presents a method for computing the remaining active gains once the entries of the gain matrix that are to be suppressed to zero are chosen. However, the choice of gains that are constrained to be zero is determined by computer simulation of many different feedback structures. It would be more desirable to obtain a mathematical approach for choosing the gains that are to be eliminated.

Calvo-Ramon[2] chooses to eliminate those gains that have the smallest influence on the closed-loop eigenvalues. This is done by computing the partial derivatives of the eigenvalues with respect to the gains and then computing the expected shift in the eigenvalues if the ijth gain were eliminated. However, Ref. 2 does not consider the effect of gain suppression on the eigenvectors. Since the eigenvectors are important for mode decoupling, the effect of gain suppression on the eigenvectors must be determined.

In this paper, expressions are derived for the partial derivatives of the eigenvectors with respect to the feedback gains. Then, these derivatives are used to compute the expected shift in the eigenvectors if the ijth feedback gain were eliminated. This establishes an improved design method for eigenstructure assignment with gain suppression by a priori choosing to eliminate those gains that have the smallest influence on both the eigenvalues and eigenvectors. An example of a fighter aircraft is presented that illustrates the importance of using both eigenvalue and eigenvector derivative information when choosing which feedback gains should be eliminated.

Control Law Development

Consider an aircraft modeled by the linear time-invariant matrix differential equation described by

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

where x is the state vector ($n \times 1$), u the control vector ($m \times 1$), and y the output vector ($r \times 1$). It is assumed that the m inputs and the r outputs are independent. Also, as is usually the case in aircraft problems, it is assumed that $m < r < n$. If there are no pilot commands, the feedback control vector u equals a matrix times the output vector y:

$$u = -Fy \tag{3}$$

The feedback problem can be stated as follows[1]: Given a set of desired eigenvalues, (λ_i^d), $i = 1,2,\ldots,r$, and a corresponding set of desired eigenvectors, (v_i^d), $i = 1,2,\ldots,r$, find a real $m \times r$ matrix F such that the eigenvalues of $A - BFC$ contain (λ_i^d) as a subset, and the corresponding eigenvectors of $A - BFC$ are close to the respective members of the set (v_i^d).

Srinathkumar[3] has shown that the feedback gain matrix F will exactly assign r eigenvalues. It will also assign the corresponding eigenvectors, provided that v_i^d is chosen to be in the subspace spanned by the columns of $(\lambda_i I - A)^{-1}B$ for $i = 1,2,\ldots,r$. This subspace is of dimension m, which is the number of independent control variables. In general, a chosen or desired eigenvector v_i^d will not reside in the prescribed subspace and, hence, cannot be achieved. Instead, a "best possi-

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ble" choice for an achievable eigenvector is made.[1] This best possible eigenvector is the projection of v_i^d onto the subspace spanned by the columns of $(\lambda_i I - A)^{-1}B$.

In many practical situations, complete specification of v_i^d is neither required nor known, but rather the designer is interested only in certain elements of the eigenvector. Thus, assume that v_i^d has the following structure[1]:

$$v_i^d = \left[v_{i1},\ x,\ x,\ x,\ x,\ v_{ij},\ x,\ x,\ v_{in}\right]^T$$

where v_{ij} are designer-specified components and x is an unspecified component. Define, as in Ref. 1, a reordering operation $\{\cdot\}^{R_i}$ such that

$$\{v_i^d\}^{R_i} = \begin{bmatrix} l_i \\ d_i \end{bmatrix} \tag{4}$$

where l_i is a vector of specified components of v_i^d and d_i is a vector of unspecified components of v_i^d. The rows of the matrix $(\lambda_i I - A)^{-1}B$ are also reordered to conform with the reordered components of v_i^d. Thus,

$$\left\{(\lambda_i I - A)^{-1}B\right\}^{R_i} = \begin{bmatrix} \tilde{L}_i \\ D_i \end{bmatrix} \tag{5}$$

Then, as shown in Ref. 1, the achievable eigenvector v_i^a is given by

$$v_i^a = (\lambda_i I - A)^{-1}B\tilde{L}_i^\dagger l_i \tag{6}$$

where $(\cdot)^\dagger$ denotes the appropriate pseudoinverse of $(\cdot)$.

The output feedback gain matrix using eigenstructure assignment is shown in Ref. 1 to be described by

$$F = -(Z - A_1 V)(CV)^{-1} \tag{7}$$

where A_1 is a partition of the matrix A in Eq. (1), V is the matrix whose columns are the r achievable eigenvectors, Z is a matrix whose columns are related to the r achievable eigenvalues and eigenvectors, and C is the output matrix in Eq. (2). The result of Eq. (7) assumes that the system described by Eqs. (1) and (2) has been transformed into a system in which the control distribution matrix B is a lead block identity matrix.

Using the development in Ref. 1, define

$$\Omega = CV \tag{8}$$

$$\psi = Z - A_1 V \tag{9}$$

Then the expression for the feedback gain matrix F is given by

$$F = -\psi\Omega^{-1} \tag{10}$$

By using the Kronecker product and the lead block identity structure of the matrix B, Ref. 1 shows that each row of the feedback gain matrix can be computed independently of all the other rows. Let ψ_i be the ith row of the matrix ψ. Then the solution for f_i, which is the ith row of the feedback gain matrix F, is given by

$$f_i = -\psi_i\Omega^{-1} \tag{11}$$

If f_{ij} is chosen to be constrained to zero, then Ref. 1 shows that f_{ij} should be deleted from f_i and that the jth row of Ω should be deleted. Let $\tilde{\Omega}$ be the matrix Ω with its jth row deleted and $\tilde{f}_i$ be the row vector f_i with its jth entry deleted. Then by using a pseudoinverse, the solution for $\tilde{f}_i$, whose entries are the remaining active gains in the ith row of the matrix F, is given by

$$\tilde{f}_i = -\psi_i\tilde{\Omega}^\dagger \tag{12}$$

where $(\cdot)^\dagger$ denotes the appropriate pseudoinverse of $(\cdot)$. If more than one gain in a row of F is to be set to zero, the $\tilde{f}_i$ and $\tilde{\Omega}$ must be appropriately modified.

Calvo-Ramon[2] proposes a method for choosing a priori which gains should be set to zero based on the sensitivities of the eigenvalues to changes in the feedback gains. The first-order sensitivity of the hth eigenvalue to changes in the ijth entry of the matrix F is denoted by $\partial\lambda_h/\partial f_{ij}$. The expected shift in the eigenvalue λ_h when constraining feedback gain f_{ij} to zero is given by[2]

$$s_{ij}^h = (f_{ij})\frac{\partial\lambda_h}{\partial f_{ij}} \tag{13}$$

Next, Ref. 2 combines all the eigenvalue shifts that are related to the same feedback gain f_{ij} to form a decision matrix $D^\lambda = \{d_{ij}\}$, $D^\lambda \epsilon R^{m\times r}$, where

$$d_{ij}^\lambda = \frac{1}{n}\left[\sum_{h=1}^{n}(s_{ij}^h)^2\right]^{1/2} \tag{14}$$

This last equation is Eq. (19) in Ref. 2. However, note that since s_{ij}^h may be complex, the d_{ij} should really be computed by using the following:

$$d_{ij}^\lambda = \frac{1}{n}\left[\sum_{h=1}^{n}(\overline{s_{ij}^h})(s_{ij}^h)\right]^{1/2} \tag{15}$$

where $(\bar{\cdot})$ denotes the complex conjugate of $(\cdot)$.

The decision matrix D^λ is used to determine which feedback gains f_{ij} should be set to zero. If d_{ij}^λ is "small," then setting f_{ij} to zero will have a small effect on the closed-loop eigenvalues. Conversely, if d_{ij}^λ is "large," then setting f_{ij} to zero will have a significant effect on the closed-loop eigenvalues. The control system designer must determine which d_{ij}^λ are "small" and which are "large" for a particular problem. In this regard, it is assumed that the states, inputs, and outputs are scaled such that these variables are expressed in the same or equivalent units.

The decision matrix D^λ is used in Ref. 2 to design a constrained output feedback controller using eigenstructure assignment. However, the sensitivities of the eigenvectors with respect to the gains are not considered when deciding which feedback gains should be set to zero. Recall that the eigenvalues determine transient response characteristics such as overshoot and settling time, whereas the eigenvectors determine mode decoupling. This mode decoupling is related, for example, to the $|\phi/\beta|$ ratio in an aircraft lateral dynamics problem. This ratio must be small, as specified in Ref. 4, which implies that the closed-loop aircraft should exhibit a significant degree of decoupling between the dutch roll mode and the roll mode. The approach of Ref. 2 may yield a constrained controller with acceptable overshoot and settling time, but the mode decoupling may be unacceptable. Thus, consideration of both eigenvalue and eigenvector sensitivities is important when choosing which feedback gains should be constrained to be zero.

The results of Ref. 2 are now extended to include both eigenvalue and eigenvector sensitivities to the feedback gains. To begin the development, consider the following result on eigenvalue and eigenvector derivatives that is credited to Lim et al.[5]

Theorem[5]: Let

$$Av_h = \lambda_h v_h; \qquad h = 1,\ldots,n$$

and

$$w_q^T A = \lambda_q w_q^T; \qquad q = 1,\ldots,n$$

Define

$$t_h = v_h^T v_h; \qquad h = 1,\ldots,n$$

and

$$s_h = w_h^T v_h; \qquad h = 1,\ldots,n$$

Then

$$\frac{\partial \lambda_h}{\partial \rho} = \left(\frac{1}{s_h}\right)\left[w_h^T\left(\frac{\partial A}{\partial \rho}\right)v_h\right] \tag{16}$$

where ρ is a scalar parameter. Also, let

$$\frac{\partial v_h}{\partial \rho} = \sum_{m=1}^{n} \alpha_{hm} v_m \tag{17}$$

Then

$$\alpha_{hq} = -\frac{1}{(\lambda_q - \lambda_h)s_q}\left[w_q^T\left(\frac{\partial A}{\partial \rho}\right)v_h\right]; \qquad q \neq h \tag{18}$$

$$\alpha_{hh} = \left(-\frac{1}{t_h}\right)\sum_{\substack{m=1\\ m\neq h}}^{n} \alpha_{hm} v_m^T v_h; \qquad q = h \tag{19}$$

Equation (16) describes the derivatives of the eigenvalues of the matrix A with respect to a scalar parameter ρ, whereas Eq. (17) describes the derivatives of the eigenvectors of the matrix A with respect to the scalar parameter ρ. The theorem will now be extended so that it applies to the matrix $A - BFC$ that describes the dynamics of Eqs. (1) and (2) when subjected to the feedback control law given by Eq. (3).

Corollary: Let

$$(A - BFC)v_h = \lambda_h v_h; \qquad h = 1,\ldots,n$$

$$w_q^T(A - BFC) = \lambda_q w_q^T; \qquad q = 1,\ldots,n$$

Define

$$t_h = v_h^T v_h; \qquad h = 1,\ldots,n$$

$$s_h = w_h^T v_h; \qquad h = 1,\ldots,n$$

Let f_{ij} be the ijth entry of the matrix F, b_i the ith column of the matrix B, and c_j^T the jth row of the matrix C. Then the derivative of the hth eigenvalue of $(A - BFC)$ with respect to the ijth element of F is given by

$$\frac{\partial \lambda_h}{\partial f_{ij}} = \left(\frac{1}{s_h}\right)(-w_h^T b_i c_j^T v_h) \tag{20}$$

and the derivative of the hth eigenvector of $A - BFC$ with respect to the ijth element of F is given by

$$\frac{\partial v_h}{\partial f_{ij}} = \sum_{m=1}^{n} \alpha_{ijhm} v_m \tag{21}$$

where

$$\alpha_{ijhq} = -\frac{1}{(\lambda_q - \lambda_h)s_q}(-w_q^T b_i c_j^T v_h); \qquad q \neq h \tag{22}$$

$$\alpha_{ijhh} = \left(-\frac{1}{t_h}\right)\sum_{\substack{m=1\\ m\neq h}}^{n} \alpha_{ijhm} v_m^T v_h; \qquad q = h \tag{23}$$

Outline of Proof: Replace $w_q^T(\partial A/\partial\rho)v_h$ by $w_h^T[\partial(A - BFC)/\partial f_{ij}]v_h$. Note that $\partial(A - BFC)/\partial f_{ij} = -B(\partial F/\partial f_{ij})C$, and after some matrix manipulations, obtain $-B(\partial F/\partial f_{ij})C = -b_i c_j^T$. Thus,

$$w_q^T\left[\frac{\partial(A - BFC)}{\partial f_{ij}}\right]v_h = -w_q^T b_i c_j^T v_h \tag{24}$$

The corollary then follows, upon using Eq. (24) together with the theorem of Ref. 5.

Remark: In the example to be presented, the eigenvectors v_h are normalized to unit norm. Then the vectors w_h^T are computed as the rows of the inverse of the normalized eigenvector matrix. Thus, for the example, the normalization factors s_h are unity. The normalization factors t_h, which may be complex, are computed by using $t_h = v_h^T v_h$.

The eigenvalue derivatives described by Eq. (20) are equivalent to the derivatives used by Calvo-Ramon.[2] However, the eigenvector derivatives, described by Eqs. (21–23), are now available. The improved approach to gain suppression is to first calculate an eigenvalue decision matrix, denoted by D^λ, by using Eqs. (13), (15), and (20). Then the eigenvector derivatives are computed by using Eqs. (21–23). These are used to compute the expected shift in eigenvector v_h when constraining feedback gain f_{ij} to be zero, which is given by [compare with Eq. (13)]

$$\bar{s}_{ij}^h = (f_{ij})\left(\frac{\partial v_h}{\partial f_{ij}}\right) \tag{25}$$

Finally, all the eigenvector shifts that are related to the same feedback gain f_{ij} are combined to form an eigenvector decision matrix D^v, where [compare with Eq. (15)]

$$d_{ij}^v = \frac{1}{n}\left[\sum_{h=1}^{n}(\bar{s}_{ij}^h)^*(\bar{s}_{ij}^h)\right]^{1/2} \tag{26}$$

where $(\cdot)^*$ denotes the complex conjugate transpose of $(\cdot)$. The gains that should be set to zero are determined by first eliminating those f_{ij} corresponding to entries of D^λ that are considered to be small. Then those entries of D^v corresponding to those f_{ij} that were chosen to be set to zero based on D^λ are reviewed. In this way, the designer can determine whether some of the f_{ij} that may be set to zero based on eigenvalue considerations should not be constrained based on eigenvector considerations.

Example

Consider the lateral directional dynamics of the F-18 HARV aircraft linearized at a Mach number of 0.38, an altitude of 5000 ft, and an angle of attack of 5 deg. The aerodynamic model is augmented with first-order actuators and a yaw rate washout filter. The eight state variables are aileron deflection δ_a, stabilator deflection δ_s, rudder deflection δ_r, sideslip angle β, roll rate p, yaw rate r, bank angle ϕ, and washout filter state x_8. The three control variables are aileron command δ_{ac}, stabilator command δ_{sc}, and rudder command δ_{rc}. The four measurements are r_{wo}, p, β, ϕ where r_{wo} is the washed out yaw rate. All quantities are in the body axis frame of reference with units of degrees or degrees per second. The state space matrices A, B, and C that completely describe the model are shown in the Appendix.

An unconstrained output feedback gain matrix is now computed by using eigenstructure assignment. The desired dutch roll eigenvalues are chosen to have a damping ratio of 0.707 and a natural frequency in the vicinity of 3 rad/s. The roll subsidence and spiral modes are chosen to be merged into a complex roll mode, as suggested in Ref. 1. The desired eigenvalues are

Dutch roll mode:

$$\lambda_{dr}^d = -2 \pm j2$$

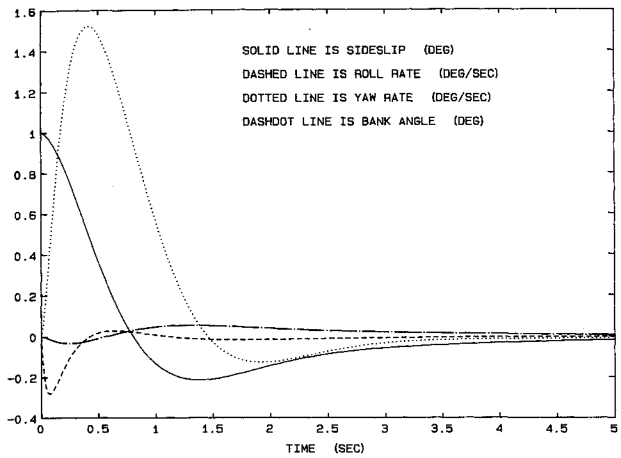

Fig. 1 Closed-loop state responses (unconstrained feedback).

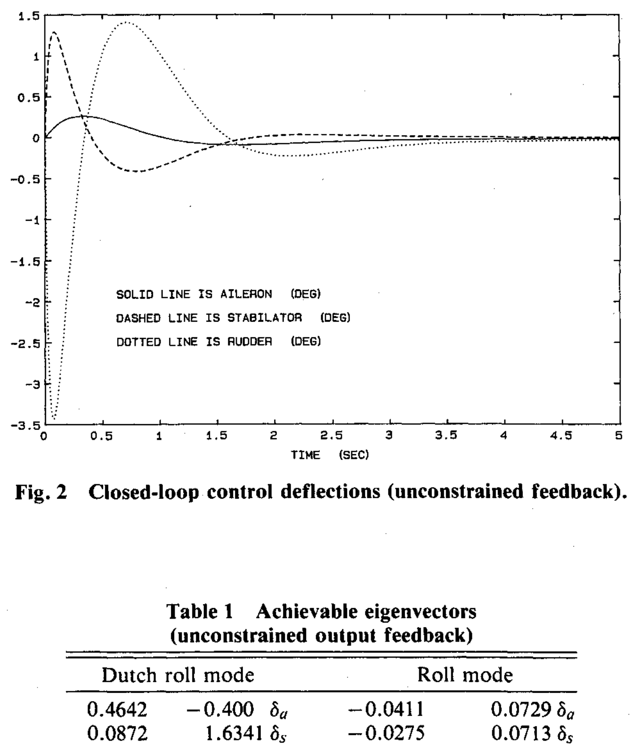

Fig. 2 Closed-loop control deflections (unconstrained feedback).

Table 1 Achievable eigenvectors (unconstrained output feedback)

Dutch roll mode		Roll mode	
0.4642	$-0.400\ \delta_a$	-0.0411	$0.0729\ \delta_a$
0.0872	$1.6341\ \delta_s$	-0.0275	$0.0713\ \delta_s$
0.6703	$-5.4587\ \delta_r$	-0.0199	$-0.0285\ \delta_r$
$0.9928 \pm j$	$0.0000\ \beta$	$\underline{-0.0175} \pm j$	$\underline{-0.0077}\ \beta$
$\underline{-0.0180}$	$\underline{0.0219}\ p$	0.9996	$0.0000\ p$
1.8396	$-2.2402\ r$	$\underline{0.0047}$	$\underline{-0.0012}\ r$
$\underline{-0.0793}$	$\underline{0.0078}\ \phi$	-0.2308	$-0.1538\ \phi$
-0.5792	$-0.0255\ x_8$	$\underline{-0.0007}$	$\underline{-0.0003}\ x_8$

Roll mode:

$$\lambda^d_{\text{roll}} = -3 \pm j2$$

The desired eigenvectors are chosen to keep the quantity $|\phi/\beta|$ small. Therefore, the desired dutch roll eigenvectors will have zero entries in the rows corresponding to bank angle and roll rate. The desired roll mode eigenvectors will have zero entries in the rows corresponding to yaw rate, sideslip, and x_8 (which is filtered yaw rate). The desired eigenvectors are

$$v^d_{dr} = \begin{bmatrix} x \\ x \\ x \\ 1 \\ 0 \\ x \\ 0 \\ x \end{bmatrix} \pm j \begin{bmatrix} x \\ x \\ x \\ x \\ 0 \\ x \\ 0 \\ x \end{bmatrix} \qquad v^d_{\text{roll}} = \begin{bmatrix} x \\ x \\ x \\ 0 \\ 1 \\ 0 \\ x \\ 0 \end{bmatrix} \pm j \begin{bmatrix} x \\ x \\ x \\ 0 \\ x \\ 0 \\ x \\ 0 \end{bmatrix} \begin{matrix} \delta_a \\ \delta_s \\ \delta_r \\ \beta \\ p \\ r \\ \phi \\ x_8 \end{matrix}$$

The achievable eigenvectors are computed using Eq. (6). However, care must be taken when computing the pseudoinverse of $\bar{L}_i$ because this matrix is ill-conditioned. Reference 6 (Sec. 2.9, pp. 52–64) suggests computing the pseudoinverse by using a singular value decomposition in which the singular values that are significantly smaller than the largest singular value are treated as zero. The achievable eigenvectors in this paper were computed on an IBM PC-XT using PC-MATLAB[7] function PINV with TOL = 0.01. The achievable eigenvectors are shown in Table 1 where the underlined numbers indicate the small couplings between p, ϕ, and the dutch roll mode and between β, r, x_8, and the roll mode. Hence, the ratio $|\phi/\beta|$ can be expected to be small.

The feedback gain matrix is computed using Eq. (7) and is shown in Table 2. The closed-loop state and control deflection responses to an initial sideslip angle of 1 deg are shown in Figs. 1 and 2, respectively. Observe that the maximum absolute values of the bank angle and roll rate are 0.056 deg and 0.287 deg/s, respectively.

The eigenvalue decision matrix D^λ and the eigenvector decision matrix D^v are shown in Table 3. The entries of D^λ that are considered large are underlined in Table 3. Observe that only 7 of the 12 feedback gains are needed when only eigenvalue sensitivities are considered. The constrained output feedback gain matrix based on using only the information available from the eigenvalue decision matrix D^λ is shown in Table 2. The state responses to an initial sideslip angle of 1 deg are shown in Fig. 3. Observe the significantly increased coupling between sideslip and bank angle. The maximum absolute values of the bank angle and roll rate are now 0.178 deg and 1.028 deg/s compared with 0.056 deg and 0.287 deg/s that were obtained with the unconstrained feedback gain matrix. The

Table 2 Comparison of control laws

	Feedback gain matrix, deg/deg				Gain and phase margins	
	r_{wo}	p	β	ϕ	(at inputs δ_{ac}, δ_{sc}, δ_{rc})	Max$\vert\phi\vert$, max$\vert p\vert$
Unconstrained	-0.1518	0.1370	-0.0580	$0.4128\ \delta_a$	[-5.50 dB, 19.12 dB]	0.0532 deg
	0.6941	0.1088	-1.6227	$0.4824\ \delta_s$		
	-2.2815	0.0177	4.5311	$-0.3890\ \delta_r$	± 52.41 deg	0.2815 deg/s
Constrained D^λ only	0	0.4472	0	1.7004	[-4.87 dB, 12.11 dB]	0.2044 deg
	0.3726	0.5734	0	2.3813		
	-2.2790	0	4.5534	0	± 44.17 deg	1.1082 deg/s
Constrained D^λ and D^v	-0.1633	0.1536	0	0.4807	[-5.34 dB, 16.46 dB]	0.0546 deg
	0.6941	0.1088	-1.6227	0.4824		
	-2.2790	0	4.5534	0	± 50.28 deg	0.3199 deg/s

Table 3 Eigenvalue and eigenvector decision matrices

Eigenvalue decision matrix D^λ				Eigenvector decision matrix D^ν			
r_{wo}	p	β	ϕ	r_{wo}	p	β	ϕ
0.06320	0.50784	0.00408	0.36134 δ_a	0.22807	0.56425	0.02222	0.08364 δ_a
0.22142	0.31886	0.07930	0.33325 δ_s	0.82699	0.35583	0.49243	0.07149 δ_s
0.67730	0.01660	0.33730	0.04826 δ_r	0.90978	0.03346	0.27513	0.08631 δ_r

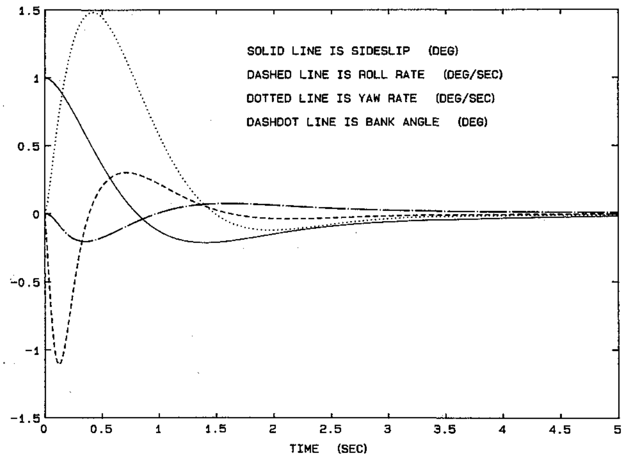

Fig. 3 Closed-loop state responses (constrained using only D^λ).

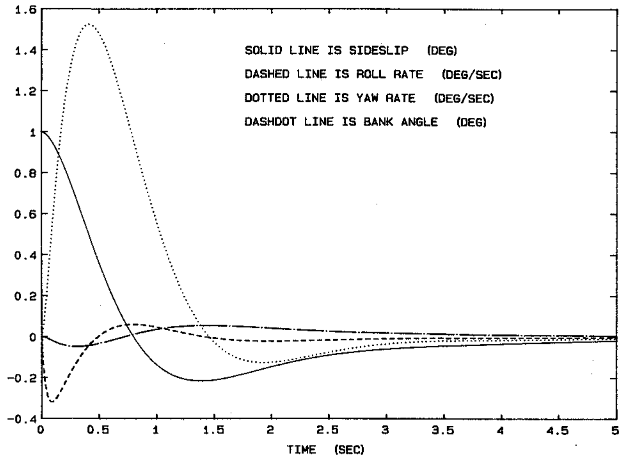

Fig. 4 Closed-loop state responses (constrained using both D^λ and D^ν).

increased coupling is due to ignoring the eigenvector sensitivities and illustrates the importance of the eigenvectors in achieving adequate mode decoupling.

Next, consider the entries of D^ν that correspond to those f_{ij} that were chosen to be set to zero based on the eigenvalue decision matrix. The two largest d_{ij}^ν that belong to this class are d_{11}^ν and d_{23}^ν. A new constrained output feedback gain matrix is computed in which f_{11} and f_{23} are not set to zero. Thus, nine gains now need to be unconstrained when using both eigenvalue and eigenvector information. The new feedback gain matrix is given in Table 2 and the state responses for an initial condition of 1-deg sideslip angle are illustrated in Fig. 4. Observe that these time responses are almost identical to the responses in Fig. 1, which were obtained by using all 12 feedback gains. Thus, a simpler controller is obtained with a negligible change in the aircraft time responses. Finally, the multivariable gain and phase margins at the inputs are computed using the results of Lehtomaki et al.[8] These margins are shown in Table 2 for each of the three feedback gain matrices. The values of these margins were accepted in light of the conservatism inherent in singular-value-based multivariable stability margin computation.

Conclusions

Eigenstructure assignment was considered with gain suppression in which selected entries in the output feedback gain matrix are eliminated. Expressions were derived for the partial derivatives of the eigenvectors with respect to the feedback gains. These derivatives were used to compute the expected shift in the eigenvectors if the ijth feedback gain were eliminated. This establishes an improved method for eigenstructure assignment with gain suppression in which both eigenvalue and eigenvector derivatives are used when selecting the feedback gains that are eliminated. An example of a fighter aircraft was presented that illustrates the importance of using both eigenvalue and eigenvector derivatives when choosing which gains are to be set to zero. A control law using only 9 of the 12 possible feedback paths was shown to exhibit time responses that are almost identical to the unconstrained control law for the example problem.

Appendix: Data for the F-18 HARV Lateral Directional Dynamics at $M = 0.38$, $H = 5000$ ft, and $\alpha = 5$ deg

$$A = \begin{bmatrix} -30.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -30.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -30.0000 & 0 & 0 & 0 & 0 & 0 \\ -0.0070 & -0.0140 & 0.0412 & -0.1727 & 0.0873 & -0.9946 & 0.0760 & 0 \\ 15.3225 & 12.0601 & 2.2022 & -11.0723 & -2.1912 & 0.7096 & 0 & 0 \\ -0.3264 & 0.2041 & -1.3524 & 2.1137 & -0.0086 & -0.1399 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0.0875 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5000 & 0 & -0.5000 \end{bmatrix}$$

$$B = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 30 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

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